The University of Georgia
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Understanding Functions Through GSP and Dynagraphs

Mathematics 1, Unit 1: Instructional Unit

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Day #1

MM1A1a: Represent functions using function notation.

Essential Question:
What is a function and how do you represent functions using function notation?

Activities:
Before introducing function notation, the students will need to be reminded of relations and functions. The students began discussing the concept of a function in middle school, but they will investigate functions at a deeper level in Math 1.

First, ask the students to try to remember everything they can about functions. Let them brainstorm for a few minutes. Depending on their responses, the teacher might need to provide the following definitions:

> **Relation**: A rule that gives an output number for every valid input number.
> **Function**: A relation in which each element of the domain (or input value) is related to *only one* element of the range (or output value).
> **Domain**: The set of x-coordinates of the set of points on a graph; the set of x-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.
> **Range**: The y-coordinates of the set of points on a graph. Also, the y-coordinates of a given set of ordered pairs. The range is the output in a function or a relation.

Show the students the Functions GSP sketch so they can see a visual representation of what a function is and some examples of relations that are and are not functions. The GSP sketch will show the various representations of a function, such as mappings, ordered pairs, tables and graphs.

Students will be introduced to function notation through the use of dynagraphs. The students will be given input and output values and will try to figure out the function. The teacher will need to make sure that the students understand how to represent functions using function notation. This GSP sketch contains a dynagraph for each parent function in Math 1.

For homework, the students will complete problems from a worksheet involving function notation. Some problems on the worksheet might be too difficult, so just pick and choose the problems the students will be able to complete. See the worksheet in the Supplement Folder for Day 1.

Day #2

MM1A1b: Graph the basic functions $f(x) = x^n$, where n=1 to 3, $f(x) = \sqrt{x}$, $f(x) = |x|$, and $f(x) = 1/x$.

Essential Question:
How do you graph the six parent functions?

Activities:
The teacher will need to review the six parent functions the students investigated the previous day: linear, quadratic, cubic, absolute value, square root, and rational. The teacher will need to make sure that the students know the equation (in function notation) for each parent function. The students will need 3 pieces of graph paper each (with grids on the front and back) so that they can graph the 6 parent functions. The students will need to graph each parent function on their graph paper.

Before the students start plotting points for the parent functions, the teacher might need to review with the students the basics of the Cartesian plane and how to plot points. For instance, the teacher can review the four different quadrants, the x and y axes, positive and negative values, etc.

Suggest to the students that it might be best to make a T-table to help plot points. Completing a table of points to plot will also give the students extra practice with working with function notation. The students may work in groups so that they can compare their work to their peer's work. When the students have finished graphing the parent functions, show the students the GSP sketch with the dynagraphs and the graphs of the parent functions. With this sketch, the students will be able to see the connection between input and output values with the graph. The sketches might also initiate ideas related to domain and range, which will be covered on Day 4 and 5 of this unit.

If the students do not complete their graphs during class, they will need to complete them for homework.

Day #3

MM1A1c: Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes.

Essential Question:
How can parent functions be changed and what does it do to the graph of the function?

Activities:
This lesson will begin with the teacher showing a GSP sketch. The sketch will contain several dynagraphs of the parent functions that have undergone some sort of transformation, such as a vertical shift up or dow, a vertical stretch, a vertical shrink, a reflection across the x-axis, or a reflection across the y-axis.

The students may try to guess the function the dynagraphs represent. This will be a good exercise for students to develop and practice their critical thinking skills in math class. The students may want to use scratch paper to jot down their thoughts and calculations.

After the students have studied the dynagraphs, show the students the function. Ask the students what's different about the function. How does it compare to the parent function. After the students have discussed the differences, plot the parent function on the GSP sketch and then plot the points from the dynagraph and trace the points. This way, the students will be able to compare the transformed function to the parent function.

Ask the students to write down how the equation of the parent function changed and connect it to how it made the graph of the parent function change. The teacher will need to describe to the students which transformation is taking place. For example, a student might say that the transformed graph got taller or skinner. The teacher would need to explain that this is a vertical stretch.

The students might want to make a chart for their notes that is similar to the one below.

Name of Transformation	What do we do to the equation?	What do we do to the graph?
Vertical Shift Up		
Vertical Shift Down		
Vertical Stretch		
Vertical Shrink		
Reflection Over X-Axis		
Reflection Over Y-Axis		

For homework, have the students transform each parent function once. Encourage the students to use one of each type of transformation that they learned about. The students will need to write the transformed function as an equation, describe what transformation is taking place, and then graph the transformed function to justify their statements. The teacher may need to provide the students with graph paper.

Day #4/5 (Activities will need to occur over two days)

MM1A1d: Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.

Essential Question:
What are the characteristics of a function and how do you recognize them?

Activities:
For this lesson, the students will investigate the characteristics of functions, but they will first need to know what the different characteristics are. The teacher may come up with their own definitions or use the definitions below:

> **Domain**: All of the input values (or x values) for a given function.
> **Range**: All of the output values (or y values) for a given function.
> **Zeros**: A value for x that makes the output value of the function 0, or where the graph of a function crosses or touches the x-axis.
> **Intercepts**: The point where a graph crosses either axes on the coordinate plane.
> **Interval of Increase**: An interval of x values for which the output values of a function increase as the input values increase.
> **Interval of Decrease**: An interval of x values for which the output values of a function decrease as the input values increase.
> **Maximum**: The largest output value a function can achieve.
> **Minimum**: The smallest output value a function can achieve.
> **Left End Behavior**: The output value the function is approaching as the input values get smaller.
> **Right End Behavior**: The output value the function is approaching as the input values get larger.

After the students know what the different characteristics are, the teacher will need to explain how to find these characteristics by either using the equation of the function or the graph of the function. The methods and suggestions below can be used to show students how to find the different characteristics. The teacher will need to pull up the GSP sketch for the day and use the linear function to describe how to find each characteristic.

> How to find…
> **Domain**: (Graphically) Determine all the x values the graph of the function covers. (Algebraically) Determine what values, if any, can be plugged in to the equation of the function.
> **Range**: (Graphically) Determine all the y values the graph of the function covers. (Algebraically) Determine the values that are the result from plugging in all the values in the domain.
> **Zeros**: Determine where the graph of the function crosses or touches the x-axis. Some functions might not have a zero.
> **Interval of Increase**: Think of a roller coaster traveling up and down the graph of the function, starting on the left and traveling to the right. The interval of increase is the interval of x values from where the roller coaster starts going uphill until it starts going downhill.
> **Interval of Decrease**: Thinking of that same roller coaster, the interval of decrease is the interval of x values from where the roller coaster starts going downhill until it starts going uphill.
> **Maximum**: Determine the biggest y value the graph of the function will eventually reach.
> **Minimum**: Determine the smallest y value the graph of the function will eventually reach.

Left End Behavior: Ask yourself, as the x values are getting smaller on my graph, what are the y values approaching?
Right End Behavior: Ask yourself, as the x values are getting bigger on my graph, what are the y values approaching?

The students will need to find the characteristics of the remaining parent functions. For the second day of instruction, the teacher will need to go over the characteristics of the remaining parent functions with the students. The teacher may use the dynagraphs of each parent function when needed to help the students see certain relationships. For example, students might find it easier to determine the domain and range of the square root function because the function disappears when the input values are negative.

For homework, the students will need to find all the characteristics of two transformed functions: $f(x) = |x| + 6$ and $g(x) = x^2 - 7$.

Day #6

MM1A1h: Determine graphically and algebraically whether a function has symmetry and whether it is even, odd, or neither.

Essential Question:
How are functions and symmetry related?

Activities:
To begin the lesson, have a brief mini-lesson/warm-up activity with transformations. The students learned about the various transformations in middle school. Let the students discuss what they remember, but focus on rotations and reflections. To show a visual representation of a reflection and rotation, open the Day 6 GSP sketch. The first tab in the document will show an example of a rotation and a reflection. The students will need to understand the concepts of reflection and rotation in order to understand symmetry.

The next GSP sketch discusses symmetry. The sketch describes y-axis symmetry and symmetry about the origin. It then provides graphs of the six parent functions. The students are to determine which graphs have symmetry or no symmetry. (The quadratic and absolute value parent graphs have y-axis symmetry. The linear, cubic, and rational parent graphs are symmetric about the origin. The square root parent graph has no symmetry.)

The remaining two tabs in the sketch uses what the students learn about symmetry and applies it to even and odd functions. One sketch shows how even functions are symmetric about the y-axis and the other sketch shows how odd functions are symmetric about the origin.

With the remaining time in class, or for homework, the students will need to take each parent function (except square root) and transform them (using the transformations they learned on Day #3) in two different ways. One transformation will need to remain an even or odd function, but the second transformation will need to be neither even nor odd. So, the first transformation will remain symmetric, but the second transformation will not be symmetric across the y-axis or about the origin.

Day #7

MM1A1e: Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

Essential Question:
How can we relate what we know about functions to a specific scenario or context?

Activities:
For this lesson, the students will be working on a learning task, Fences and Functions, from the Georgia Performance Standards Frameworks, just slightly modified and shortened. The students will be able to work collaboratively in groups and will use what they have learned previously in the unit to complete the task. Parts of the learning task require the students to graph certain relations. The teacher may graph these relationships using GSP if needed.

The students will need to complete the task for homework if they do not complete it in class.

Day #8

MM1A1g: Explore rates of change, comparing constant rates of change (i.e., slope) versus variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.

Essential Question:
What is rate of change and what does it tell us about a function?

Activities:
To begin the lesson, the teacher will need to discuss speed. Speed, in the United States, is normally written as miles per hour. Ask the students what that means. Miles per hour is essentially a distance divided by a time. Explain to the students this is how rate is calculated: r = d / t.

The students will then complete a Rate of Change worksheet. The students may work in groups to complete the worksheet. The students will need to plot two sets of data, so the teacher may also want to plot the data using GSP so the students can compare and check their graphs. One set of data will yield a linear graph and the other set will be non-linear. The students are to connect a constant rate of change with a linear graph and a variable rate of change with a non-linear graph. The students will also see that the slope of line is also the same as the average rate of change for the function.

For homework, the students will complete a worksheet involving word problems and rate of change. The worksheet can be found in the Supplement Folder.

Day #9

MM1A1f: Recognize sequences as functions with domains that are whole numbers.

Essential Question:
How can patterns be related to functions?

Activities:
The students will be completing another worksheet that deals with writing sequences as functions (see Supplement Folder). The students will need to understand that when a function is written to represent a sequence, the range of the function is just the sequence itself and the domain of the function represents the position in the sequence for each term. For example, with the Fibonacci Sequence (1, 1, 2, 3, 5, 8, 13, …), the domain is 1, 2, 3, 4, 5, … and the range is 1, 1, 2, 3, 5, 8, 13, …, where 4 in the domain maps to 3 in the range.

The teacher can show the students the graphs of the sequences on the worksheet using GSP. It will need to be emphasized that the graphs of the sequences are not continuous; graphing the sequence as a function would just result in plotted points that are not connected.

If time permits, the students might enjoy the following video on the Fibonacci Sequence:
http://www.youtube.com/watch?v=2nAyCc7sGVI

For homework, the students will need to look at the first seven sequences on their worksheet and try to write each as a function with a domain of whole numbers. Some of the sequences have no pattern, so the students will not be able to write it as a function.

Day #10

Essential Question:
Have you mastered all the standards that will be assessed on the Unit 1 assessment?

Activities:
The students will compete the Unit 1 Assessment.